Image formation in man and machines

- Lenses, diffraction and aberrations
- Rods, cones and CCD's
- Source: Foundations of Vision, Brian Wandell, Sinauer Associates, Sunderland MA, 1995
- Trucco and Verri: 2.1, 2.2.1, 2.2.2, 2.3.1

How do we see?

- Extromissive theories of vision
  - Plato - from our eyes flows a light similar to the light of the sun
  - "Therefore, when these three conditions concur, sight occurs, and the cause of sight is threefold: the light of the innate heat passing through the eyes, which is the principal cause, the exterior light kindred to our own light, which both acts and assists, and the light that flows from visible bodies, flame or color; without these the proposed effect [vision] cannot occur." [Chalcidius, middle ages].
  - The "evil eye"

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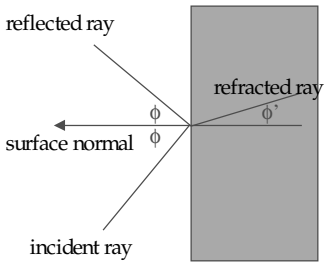
    - the soul has things timed perfectly - this never happens

How do we see?

- Intromissive theories of vision
  - objects create "material images" that are transported through the atmosphere and enter the eye (Aristotle)
    - but how do the material images of large objects enter the eye?
- Abu Ali al-Hassan ibn al-Hasan ibn al-Haytham
  - mercifully shortened to Alhazen
  - greatest optical scientist of the middle ages
    - pointillist theory of vision - we see a collection of points on the surfaces of objects
    - geometric theory to explain the 1-1 correspondence between the world and the image formed in our eyes

Lens and image formation

- Ray of light leaves the light source, and travels along a straight line
- Light hits an object and is
  - reflected and/or
  - refracted
- If the object is our lens, then the useful light for imaging is the refracted light



Ptolemy, Alhazen and refraction

- The phenomena of refraction was known to Ptolemy
- Alhazen's problem - since light from a surface point reaches the entire surface of the eye, how is it that we see only a single image of a point?
  - he assumed that only the ray that enters perpendicular to the eye effects vision
  - the other rays are more refracted, and therefore "weakened"
  - but in fact, the optical properties of the lens combine all of these rays into a single "focused" point under favorable conditions

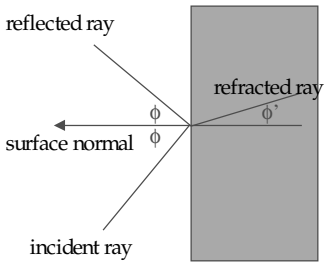
Snell's law

- If ϕ is the angle of incidence and ϕ' is the angle of refraction then

$$\frac{\sin\phi}{\sin\phi'} = \frac{\nu}{\nu'}$$

where ν and ν' are the refractive indices of the two media

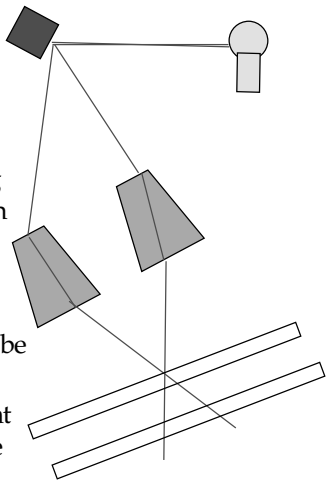
- Refractive index is the ratio of speed of light in a vacuum to speed of light in the medium



Refractive indices
glass - 1.52
water - 1.333
air - 1.000 - mercifully, or there would be no physical basis for vision!

Applying Snell's Law twice

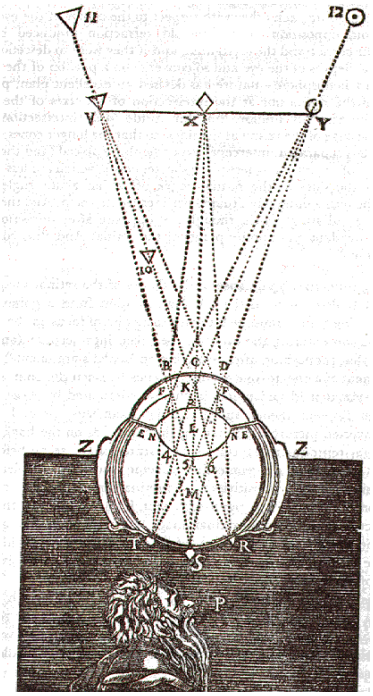
- Pass light into and out of a prism (symmetric piece of glass)
  - By combining many infinitesimally small prisms we form a convex lens that will bring all of the refracted rays incident from a given surface point into coincidence at a point behind the lens
  - If the image or film plane is placed that distance behind the lens, then that point will be in focus
  - If the image plane is in front of or behind that ideal location, the image of that point will be out of focus



Kepler's retinal theory

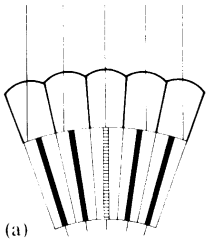
Even though light rays from "many" surface points hit the same point on the lens, they approach the lens from different directions.

Therefore, they are refracted in different directions - separated by the lens



The structure of eyes -compound eyes

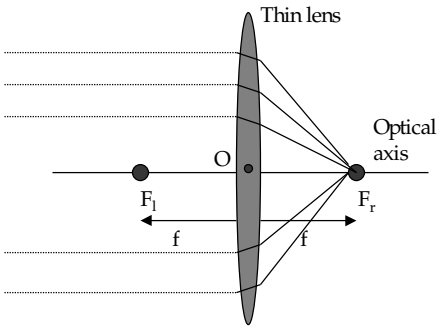
- Many (small) animals have compound eyes
  - each photoreceptor has its own lens
  - images seen by these eyes are equally sharp in all directions
  - images seen by these eyes are equally "bright" in all directions when viewing a field of constant brightness
  - examples: flies and other insects
- But these eyes do not "scale" well biologically



(a)

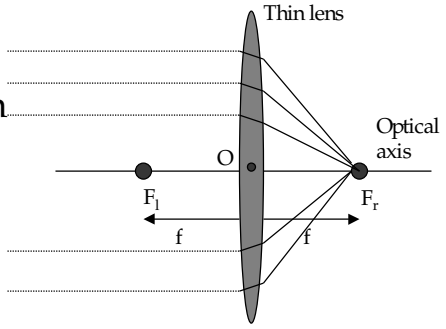
Thin lenses

- The optical behavior is determined by:
  - optical axis going through the lens center O and perpendicular to the lens center plane
  - the left and right focus (F_l and F_r) located a distance f, called the focal length, from the lens center



Thin lenses

- The shape of the lens is designed so that all rays parallel to the optical axis on one side are focused by the lens on the other side:
  - Any ray entering the lens parallel to the optical axis on one side goes through the focus on the other side.
  - Any ray entering the lens from the focus on one side, emerges parallel to the optical axis on the other side.

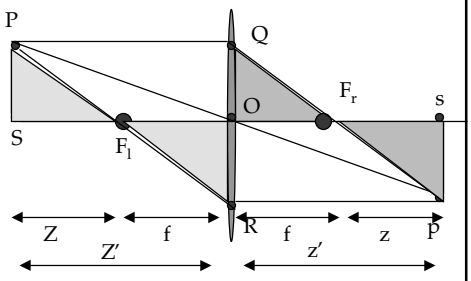


Lensmaker's equation

- Lens imperfections might cause these rays not to intersect at a point
  - deviations in shape from the ideal lens
  - material imperfections that might cause the refractive index to vary within the lens
- Lensmakers equation
  - relates the distance between the point being viewed and the lens to the distance between the lens and the ideal image (where the rays from that point are brought into focus by the lens)
  - Let P be a point being viewed that is not too far from the optical axis.
    - Z + f is the distance of P from the lens along the optical axis
    - The thin lens focuses all the rays from P onto the same point, the image point, p

Lensmakers equation $\frac{1}{Z'}+\frac{1}{z'}=\frac{1}{f}$

- p can be determined by intersecting two known rays, PQ and PR.
  - PQ is parallel to the optical axis, so it must be refracted to pass through F_r.
  - PR passes through the left focus, so emerges parallel to the optical axis.
- Note two pairs of similar triangles
  - PF_lS <-> ROF_l and psF_r <-> QOF_r



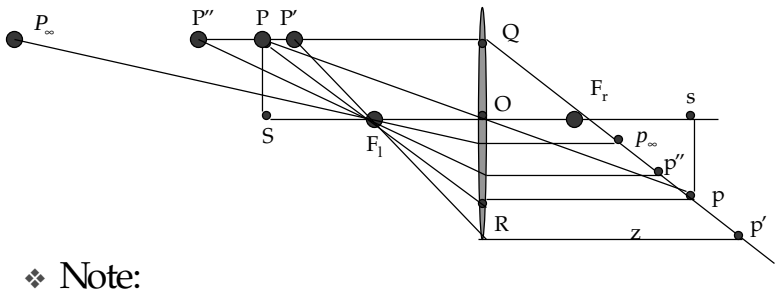
- PS/OR = Z/f
- QO/sp = f/z
- But PS = QO and OR = sp
- So, Z/f = f/z, or $Zz = f^2$
- Let Z' = Z + f and z' = z + f

$$\frac{1}{Z'}+\frac{1}{z'}=\frac{1}{f}$$

Lensmaker's equation $\frac{1}{Z'}+\frac{1}{z'}=\frac{1}{f}$

- Notice that the distance behind the lens, z′, at which a point, P, is brought into focus depends on Z′, the distance of that point from the lens
  - familiar to us from rotating the focus ring of any camera.
- In real lenses, system is designed so that all points within a given range of distances [Z_1, Z_2] are brought into focus at the same distance behind the lens center.
  - This is called the depth of field of the lens.

Lensmaker's equation $\frac{1}{Z'}+\frac{1}{z'}=\frac{1}{f}$



- Note:
  - as Z′ gets large, z′ approaches f
  - as Z′ approaches f, z′ approaches infinity

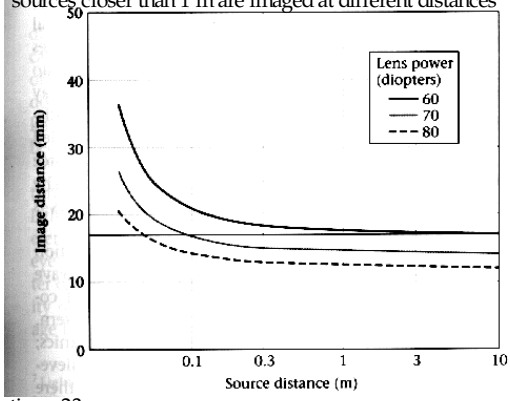
Optical power and accommodation

- Optical power of a lens - how strongly the lens bends the incoming rays
  - short focal length lens bends rays significantly
  - it images a point source at infinity (large d_s) at distance d_i behind the lens. The smaller f, the more the rays must be bent to bring them into focus sooner.
  - optical power is $1/f$, measured in meters. The unit is called the diopter
  - Human vision: when viewing faraway objects the distance from the lens to the retina is .017m. So the optical power of the eye is 58.8 diopters

Accommodation

- From the lensmakers equation and the known optical power of the eye ($1/f$) we can calculate the ideal image distance of a point being viewed as a function of the distance to that point

sources at > 1 meter are imaged at same distance
sources closer than 1 m are imaged at different distances



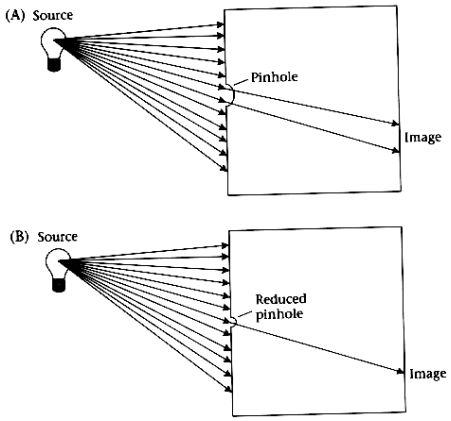
2.16 DEPTH OF FIELD OF THE HUMAN EYE. Image distance is shown as a function of source distance. The solid horizontal line shows the distance of the retina from the lens center. A lens power of 60 diopters brings distant objects into focus, but not nearby objects; to bring nearby objects into focus the power of the lens must increase. The depth of field (the distance over which objects will continue to be in reasonable focus) can be estimated from the slope of the curve.

Accommodation

- How does the human eye bring nearby points into focus on the retina?
  - by increasing the power of the lens
  - muscles attached to the lens change its shape to change the lens power
  - accommodation: adjusting the focal length of the lens
  - bringing points that are nearby into focus causes faraway points to go out of focus
  - depth-of-field: range of distances in focus
- Physical cameras - mechanically change the distance between the lens and the image plane

Lensless imaging systems - pinhole optics

- Pinhole optics focuses images
  - without lens
  - with infinite depth of field
- Smaller the pinhole
  - better the focus
  - less the light energy from any single point

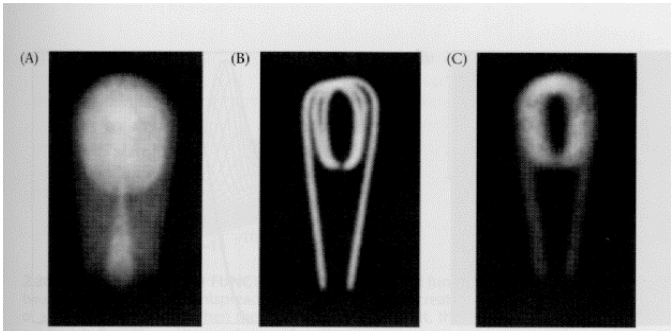


2.17 PINHOLE OPTICS. Using ray-tracing, we see that only a small pencil of rays passes through a pinhole. (A) If we use a wide pinhole, light from the source spreads across the image, making it blurry. (B) If we narrow the pinhole, only a small amount of light is let in. The image is sharp; the sharpness is limited by diffraction.

Diffraction

- Two disadvantages to pinhole systems
  - light collecting power
  - diffraction
- Diffraction
  - when light passes through a small aperture it does not travel in a straight line
  - it is scattered in many directions
  - process is called diffraction and is a quantum effect
- Human vision
  - at high light levels, pupil (aperture) is small and blurring is due to diffraction
  - at low light levels, pupil is open and blurring is due to lens imperfections

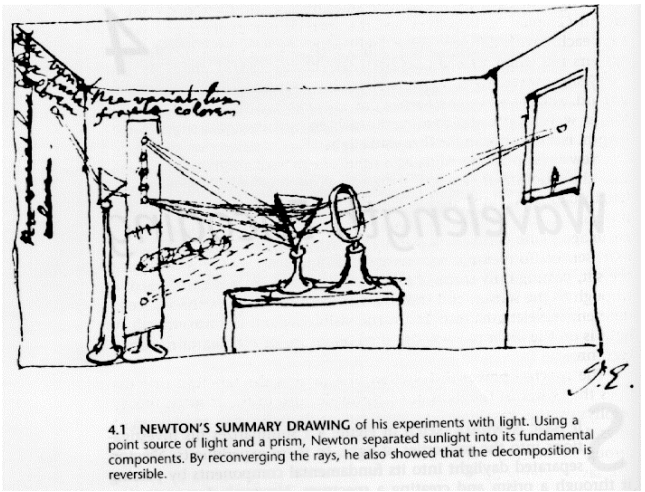
Diffraction and pinhole optics



2.18 **DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS.** These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

Complications of color

- Spectral composition of light
  - Newton's original prism experiment
  - light decomposed into its spectral components



4.1 **NEWTON'S SUMMARY DRAWING** of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

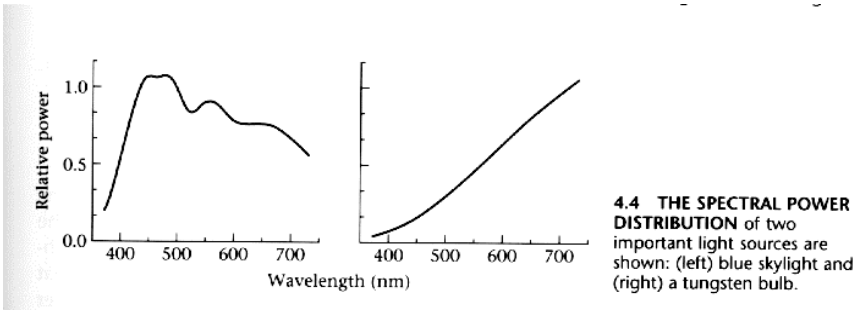
Complications of color

- Why does the prism separate the light into its spectral components?
  - prism bends different wavelengths of light by different amounts
    - refractive index is a function of wavelength
    - shorter wavelengths are refracted more strongly than longer wavelengths

Wavelength	Color (*)
700	Red
610	Orange
580	Yellow
540	Green
480	Blue
400	Violet

* - viewed in isolation

Complications of color

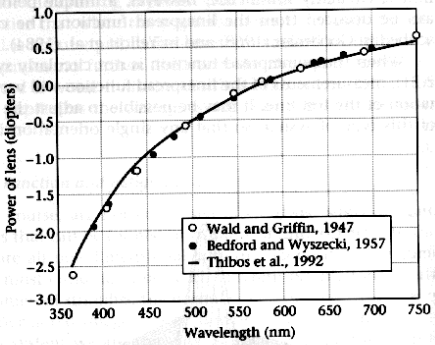


4.4 THE SPECTRAL POWER DISTRIBUTION of two important light sources are shown: (left) blue skylight and (right) a tungsten bulb.

Chromatic aberration

- Chromatic aberration
  - The prism effect of focusing different wavelengths of light from the same point source at different distances behind the lens
  - when incident light is a mixture of wavelengths, we can observe a chromatic fringe at edges
  - accommodation can bring any wavelength into good focus, but not all simultaneously
  - human visual system has other mechanisms for reducing chromatic aberration
  - color cameras have similar problems

Chromatic abberation



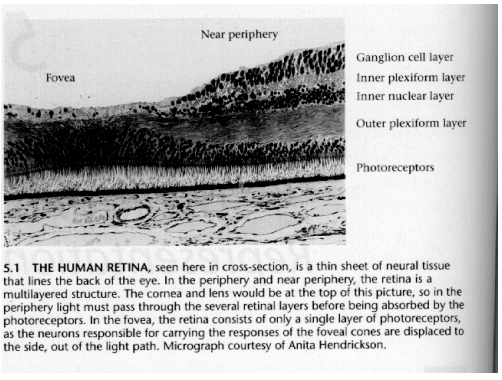
2.22 CHROMATIC ABERRATION OF THE HUMAN EYE. The data points measure the optical power one must add to the human eye in order to bring different wavelengths into a common focus with a 578-nm light. The smooth curve plots a formula created by Thibos et al. (1992) that predicts the measurements and interpolates smoothly between them. The formula is $D(\lambda) = p - q/(\lambda - c)$, where λ is wavelength in micrometers, $D(\lambda)$ is the defocus in diopters, $p = 1.7312$, $q = 0.63346$, and $c = 0.21410$. After Marimont and Wandell, 1993.

Photoreceptor mosaics

- The retina is covered with a mosaic of photoreceptors
- Two different types of photoreceptors
  - rods - approximately 100,000,000
  - cones - approximately 5,000,000
- Rods
  - sensitive to low levels of light: scotopic light levels
- Cones
  - sensitive to higher levels of light: photopic light levels
- Mesopic light levels - both rods and cones active

The human eye

- Limitations of human vision
  - Blood vessels and other cells in front of photoreceptors
  - shadows cast on photoreceptors
  - non-uniform brightness



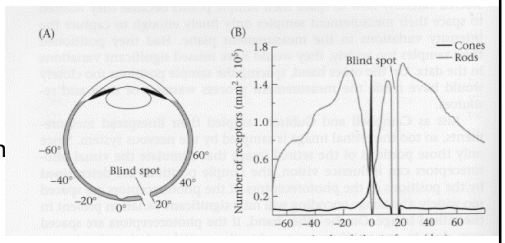
5.1 THE HUMAN RETINA, seen here in cross-section, is a thin sheet of neural tissue that lines the back of the eye. In the periphery and near periphery, the retina is a multilayered structure. The cornea and lens would be at the top of this picture, so in the periphery light must pass through the several retinal layers before being absorbed by the photoreceptors. In the fovea, the retina consists of only a single layer of photoreceptors, as the neurons responsible for carrying the responses of the foveal cones are displaced to the side, out of the light path. Micrograph courtesy of Anita Hendrickson.

Photoreceptor mosaics

- Fovea is area of highest concentration of photoreceptors
  - fovea contains no rods, just cones
  - approximately 50,000 cones in the fovea
  - cannot see dim light sources (like stars) when we look straight at them!
- TV camera photoreceptor mosaics
  - nearly square mosaic of approximately 800X640 elements for complete field of view

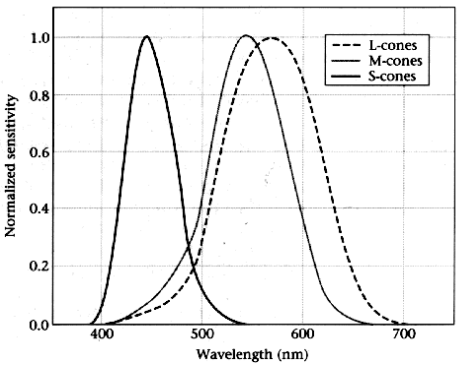
The human eye

- Limitations of human vision
  - the image is upside-down!
  - high resolution vision only in the fovea
    - only one small fovea in man
    - other animals (birds, cheetas) have different foveal organizations
  - blind spot



Cones and color

- There are three different types of cones
  - they differ in their sensitivity to different wavelengths of light



3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, AND S-CONES in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea, lens, and other inert pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.

Color cameras

- Two types of color cameras
  - Single CCD array
    - in front of each CCD element is a filter - red, green or blue
    - color values at each pixel are obtained by hardware interpolation
      - subject to artifacts
      - lower intensity quality than a monochromatic camera
      - similar to human vision
  - 3 CCD arrays packed together, each sensitive to different wavelengths of light

Cones, CCD's and space

- How much of the world does a cone see?
  - measured in terms of visual angle
  - the eye lens collects light over a total field of view of about 100^o
  - each cone collects light over a visual angle of about 1.47×10^{-4} degrees, which is about 30 seconds of visual angle
- How much of the world does a single camera CCD see
  - example: 50^o lens
  - 50/500 gives about 10^{-1} degrees per CCD